

## TRANSORB:

This utility allows GNC to calculate *low-energy transfer orbit* parameters for GUIDO and the pilot.

The software calculates how much you need to change the speed of the spacecraft while in one orbit to initiate a transfer orbit to a new circular orbit about the same reference orbit. In other words, to undertake the maneuver illustrated in the following diagram:

You also can use it to determine how much to slow down to drop down to a new lower orbit.

**The software assumes that your current orbit is circular!**

If it is not, see the note in Program Output.

### Program Input:

- 1) Current orbit reference object (what you are orbiting) from a list,
- 2) Current altitude above reference object (km from the surface),
- 3) Ending altitude above reference object (km from the surface), and
- 4) Planned engine acceleration.

### Program Output:

- 1) Required delta-V (change in velocity from ref  $V_0$  at current altitude),  
This is the amount by which your current orbital speed must be changed (- means decrease, + means increase).  
Add this amount to your current  $V_{hab-ref}$  and this is what your speed should be after your engine burn is complete.

**If your orbit is not circular, your new speed should be  $ref-V_0 + \Delta V$ .**

**If your orbit is not circular, set target to equal ref. You also will have to zero out your  $V_{cen}$  while changing  $V_{tan}$ .**

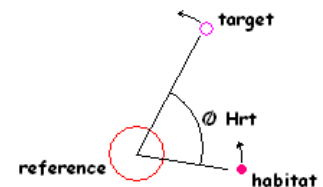
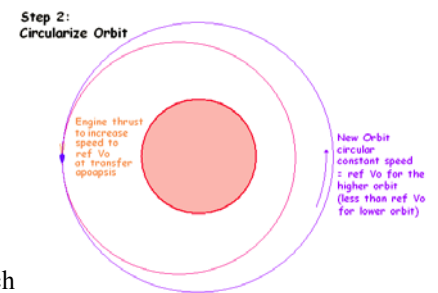
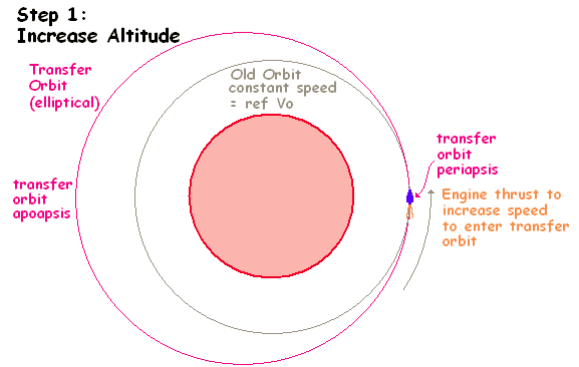
- 2) Engine burn time, assuming planned acceleration is used (seconds),  
This is for how long you will fire the engine to change your speed.
- 3) Transfer orbit duration; how long it will take to get to the target altitude (seconds),
- 4) Ref  $V_0$  of the new orbit.

This is how fast you will have to go to circularize your orbit at the new altitude.

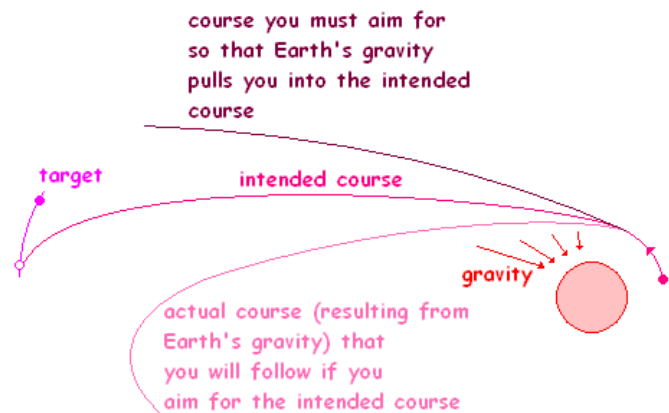
This information is not critical until after the transfer orbit is complete.

- 5) lead angle ( $\Theta_{Hrt}$ ) needed to intercept a satellite in the target orbit.

This is by what angle you should be behind (or in front, if you are dropping down) the object with which you are trying to rendezvous at the moment you change the speed. Since changing speed will take time (engine burn time), you want to start the engines such that you are at the correct  $\Theta_{Hrt}$  half-way through the engine burn time.



This software only considers the effects of the gravity of the reference object. If there are other objects with significant gravity (such as the target, or the sun) affecting the spacecraft, Transorb will only give you a rough, first approximation of the correct delta-V and lead angle. For example, if you are rendezvousing with the moon, the moon's gravity will pull the spacecraft a bit faster and cause you to overshoot your desired apoapsis. If you are moving from one planet's orbit to another, the sun is the reference object that you must use, but both your departure and destination planet will pull at the spacecraft. The departure planet, in particular, will slow you down and cause you to undershoot your desired apoapsis.





### Transorb Calculations

The basis for transorb is the gravitational equation:

$$1) F_p = \frac{GM_e m_1}{d^2}$$

$G = 6.673 \times 10^{-11} \text{ (m}^3/\text{kgs}^2\text{)}$

$M_e = 5.9742 \times 10^{24} \text{ kg}$  (the mass of the Earth, substitute the mass of what ever your orbit calculation is centered on)

$m_1$  is the mass of the object in orbit.

$d$  is the separation distance

$r_e = 6.371 \times 10^6 \text{ m}$  (the mean radius of the Earth; this must be included in any separation distance)

This equation gives the force of attraction in Newtons between the two objects.

The gravitational potential energy of the object  $m_1$  is given by the equation:

$$2) E_p = \frac{-GM_e m_1}{d}$$

$E_p$  approaches zero as the separation distance ( $d$ ) approaches infinity. Infinite separation is chosen as the zero point so that all object pairs have a common zero point for gravitational potential energy. However, since objects lose potential energy as they fall towards each other, the  $E_p$  must be negative.

### Escape Velocity

This is the speed an object (object 1) needs, relative to another object (object 2), in order to permanently escape the gravitational attraction of the other object. As object 1 moves away from the gravitational attraction of object 2, object 2's gravity continually slows down object 1. However, as equation 1 shows, this gravitational attraction will continually weaken as the separation distance increases. If object 1 is not moving away fast enough, object 2's gravity will slow object 1's speed to zero and it will start falling back towards object 2. If object 1's speed is great enough this will never happen: the gravitation attraction will weaken too rapidly, and object 1 will keep moving away.

The escape velocity of object 2 is the speed that is needed for an object moving away from object 2 to be slowed, by the gravity of object 2, to zero velocity as the separation distance approaches infinity. Any departure speed from object 2 below the escape velocity will ultimately result in a return to object 2 (unless some other object is present to attract it).

An equation for escape velocity can be derived by understanding the following:

a) Energy is conserved in a system  $E_i = E_f$

b) In 2-body gravitational systems, only kinetic energy ( $E_k$ ) and gravitational potential energy ( $E_p$ ) need be considered. The speed in question for the kinetic energy is the relative speed between the two objects.

Thus,  $(E_k + E_p)_i = (E_k + E_p)_f$

c) For two objects that start moving apart at the escape velocity, their relative speed reaches zero at infinite distance. Thus, at infinite distance,  $E_k$  will be zero; but so will be  $E_p$ . If this is our final position, then:

$$(E_k + E_p)_f = 0$$

Therefore:

$$(E_k + E_p)_i = 0 \text{ as well.}$$

d) Therefore,

$$E_{ki} = -E_{pi}$$

or:

$$\frac{1}{2} m_1 v^2 = \frac{GM_e m_1}{d}$$

where  $v$  is the escape velocity (we stated that in this situation, the separation speed started at the escape velocity)

This equation can be rearranged by dividing through by  $m_1$ , multiplying through by 2, and taking the square root of both sides to get:

$$v = \sqrt{2 \frac{GM_e}{d}}$$

This allows us to calculate the escape velocity for any planet if we know its mass and the distance that we are escaping from.

What transorb does is to calculate the relative speed needed so that object 2 slows your departing spacecraft to zero relative speed not at a distance equal to infinity, but at some much closer speed. The equations used can be derived by performing a little integral calculus on the gravitational force equation. A reasonable couple of examples can be seen by looking up “escape velocity” on wikipedia.

The program calculates how to get from altitude<sub>1</sub> above the object you are orbiting to altitude<sub>2</sub>. The radius, R, of the object you are orbiting must be added to each of these altitudes. The mass of the object that you are orbiting is M.

$$r_1 = \text{altitude}_1 + R$$

$$r_2 = \text{altitude}_2 + R$$

Keep in mind that you are currently orbiting an object, so you already have an orbital speed. What transorb calculates is how much this speed needs to change ( $\Delta v$ ) in order to effect a low energy transfer orbit from altitude 1 to altitude 2 (i.e., using 1 engine burn and coasting the rest of the way).  $\Delta v$  can be calculated by:

$$\Delta v = \sqrt{\frac{GM}{r_1}} \times \left( \sqrt{\left(\frac{2r_2}{r_1}\right) \div \left(\frac{r_2}{r_1} + 1\right)} - 1 \right)$$

The time that it takes to complete the transfer orbit is calculated by:

$$T = \pi \sqrt{\frac{(r_1 + r_2)^3}{8GM}}$$

The lead angle is the angle that you must be behind or in front of an object with which you are trying to rendezvous at the new orbital altitude. The angle is measured between both the spacecraft and the other object with the center of the orbit as the vertex of the angle.

$$\theta = \left| 360\left(\frac{T}{k}\right) - 180 \right| \quad k = 2\pi \sqrt{\frac{r_2^3}{GM}}$$

The angle is a trailing angle if  $r_1 < r_2$  and a leading angle if  $r_1 > r_2$

Once at the “top” of the transfer orbit, the spacecraft must be accelerated to the correct orbital speed for that altitude. This speed will be displayed by the spacecraft piloting software, but transorb will display this as well, calculated by:

$$v' = \frac{2\pi r_2}{k}$$

As noted earlier, what transorb cannot do, and which I do not know how to do by computation, is calculate the correction in speed and engine burn initiation point for the gravitational attraction of a third object (such as departing from the Earth when calculating a transfer orbit from the orbit of Earth around the Sun to the orbit of Mars around the sun). In the example just given, not only will Earth initially slow you down and bend your track in one direction, but Mars will speed you up and bend your track in the other direction at the end of the transfer orbit. These corrections must be worked out by simulation.